Math 70 : 6.3, 6.4, 6.5 Factoring

Lesson 18

6.3 Factoring Trinomials of form ax^2+bx+c $(a \neq \pm 1)$

Objectives

1) Factor Trinomials
   - guess + check by FOIL
   - rewrite and group "the ac method" or X

6.4 Factoring Perfect Square Trinomials & Differences of Squares

Objectives

1) Factor perfect square trinomials

$$A^2+2AB+B^2=(A+B)^2$$
$$A^2-2AB+B^2=(A-B)^2$$

2) Factor differences of squares

$$A^2-B^2=(A+B)(A-B)$$

3) Factor by grouping with factors that can be factored again.

6.5 Factoring Sums or Differences of Cubes

Objectives

1) Factor sum of cubes

$$A^3+B^3=(A+B)(A^2-AB+B^2)$$

2) Factor difference of cubes

$$A^3-B^3=(A-B)(A^2+AB+B^2)$$

Math 70: 6.3, 6.4 & 6.5 Factoring Non-Unitary Trinomials and Special Patterns

Objectives

1) Factor non-unitary trinomials
   a. Guess-and-check using FOIL
   b. Double X or ac method
2) Perfect Square trinomials
3) Differences of Squares: $a^2 - b^2 = (a-b)(a+b)$
4) Sum of Squares $a^2 + b^2$ is prime
5) Sum of Cubes $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ -- signs by SOAP
6) Difference of Cubes $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$ -- signs by SOAP

Factor completely.

1) $2x^2 + 11x + 15$

2) $m^2 - 10m + 25$

3) $16y^2 + 49 + 56y$

4) $-20xy + 4y^2 + 25x^2$

5) $32 - 162t^4$

6) $32 + 162t^4$

7) $(x+3)^2 - 36$

8) $-49z^6 + 81z^4$

9) $5 - 5x^2y^6$

10) $x^3 + 3x^2 - 4x - 12$

11) $y^3 - 64$

12) $8x^3 + 27y^3$

13) $375q^2 + 3n^3q^2$

14) $r^6 - s^6$

Math 70 6.3, 6.4, 6.5

Factor completely

1. $2x^2 + 11x + 15$

Guess and check Method

Step 1: factors of $2x^2$

x and $2x$

or $2x$ and x

factors of 15

1 and 15

3 and 5

5 and 3

15 and 1

step 2:
Assemble the combinations mentally or on paper

$(x+1)(2x+15)$
$(x+3)(2x+5)$
$(x+5)(2x+3)$
$(x+15)(2x+1)$

$(2x+1)(x+15)$
$(2x+3)(x+5)$
$(2x+5)(x+3)$
$(2x+15)(x+1)$

step 3
Check by FOIL — we have ensured $2x^2$ and 15, check only the Inside and Outside of FOIL to get the middle term.

$(x+1)(2x+15)$ $\quad 2x + 15x = 17x$ not $11x$ reject

$\boxed{(x+3)(2x+5)}$ $\quad 6x + 5x = 11x$ yes!

ac method or double x method

step 1 write in standard form $2x^2 + 11x + 15$
multiply $a \cdot c$ $\quad ax^2 + bx + c$

$2 \cdot 15$

11

b

Step 2: find two numbers which multiply to ac but add to b.

30

6 5

11

$6 \cdot 5 = 30$

$6 + 5 = 11$

CAUTION! These aren't the answer

Math 70

step 3: Rewrite the middle term of the question as two like terms with the coefficients found from X. Order doesn't matter.

$2x^2 + 11x + 15$ $\quad$ or $\quad$ $2x^2 + 11x + 15$

$2x^2 + 5x + 6x + 15$ $\quad$ or $\quad$ $2x^2 + 6x + 5x + 15$

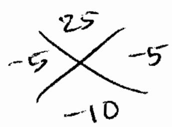
step 4 Factor the resulting four terms. using grouping.

$x(2x+5) + 3(2x+5)$ $\quad$ or $\quad$ $2x(x+3) + 5(x+3)$

$\boxed{(2x+5)(x+3)}$ $\quad$ or $\quad$ $\boxed{(x+3)(2x+5)}$.

(2) $m^2 - 10m + 25$

Method 1



$\Rightarrow \boxed{(m-5)(m-5)}$

or $\boxed{(m-5)^2}$

Method 2: Notice perfect squares

$m^2 - 10m + 25 = \boxed{(m-5)^2}$

$(m)^2 \quad 2(-5m) \quad (5)^2$

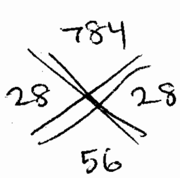
(3) $16y^2 + 49 + 56y$

CAUTION! Problem is not in standard form!

$16y^2 + 56y + 49 = \boxed{(4y+7)^2}$

$(4y)^2 \quad 2(4y \cdot 7) \quad (7)^2$

Can be done using guess and check or X



$\underbrace{16y^2 + 28y} + \underbrace{28y + 56}$

$4y(4y+7) + 7y(4y+7)$

$\boxed{(4y+7)(4y+7)}$

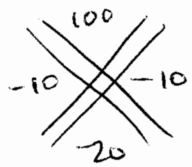
Math 70

(4) $-20xy + 4y^2 + 25x^2$

CAUTION! Not in standard form!

$$\underset{\uparrow\atop (5x)^2}{25x^2} - \underset{\uparrow\atop -2(5x)(2y)}{20xy} + \underset{\uparrow\atop (2y)^2}{4y^2} = \boxed{(5x-2y)^2}$$

or X method



$$25x^2 \underbrace{-10xy - 10xy}\, + 4y^2$$

notice: like terms $-10xy - 10xy = -20xy$
match original question middle term

Factor by grouping

$$5x(5x-2y) - 2y(5x-2y)$$

$$= \boxed{(5x-2y)(5x-2y)}$$

or $\boxed{(5x-2y)^2}$

(5) $32 - 162t^4$

GCF: $2(16-81t^4)$

difference of squares

$$2\underbrace{(4-9t^2)}\underbrace{(4+9t^2)}$$

another difference of squares

sum of squares is prime

$$\boxed{2(2-3t)(2+3t)(4+9t^2)}$$

or

standard form

$-162t^4 + 32$

GCF $-2(81t^4 - 16)$

difference of squares

$$-2\underbrace{(9t^2-4)}\underbrace{(9t^2+4)}$$

another difference of squares

sum of squares is prime

$$\boxed{-2(3t-2)(3t+2)(9t^2+4)}$$

(6) $32 + 162t^4$

$$\boxed{2(16+81t^4)}$$

sum of squares is prime

or

standard form

$162t^4 + 32$

$$= \boxed{2(81t^4+16)}$$

Math 70

(7) $(x+3)^2 - 36$

difference of squares $a^2 - b^2$ where $a = (x+3)$

$b = 6$.

$= (x+3-6)(x+3+6)$

$= \boxed{(x-3)(x+9)}$

OR - - - - the long way....

$(x+3)(x+3) - 36$

$= x^2 + 6x + 9 - 36$

$= x^2 + 6x - 27$

-27

9 -3

6

$= \boxed{(x+9)(x-3)}$

(8) $-49z^6 + 81z^4$

GCF: $-z^4(49z^2 - 9)$

difference of squares

$= \boxed{-z^4(7z-3)(7z+3)}$

(9) $5 - 5x^2y^6$

GCF $5(1 - x^2y^6)$

difference of squares

$\boxed{5(1-xy^3)(1+xy^3)}$

or

standard form

$-5x^2y^6 + 5$

GCF $-5(x^2y^6 - 1)$

diff of sq $\boxed{-5(xy^3-1)(xy^3+1)}$

(10) $x^3 + 3x^2 - 4x - 12$

4 terms → grouping

$= x^2(x+3) - 4(x+3)$

$= (x+3)(x^2-4)$ diff of sq.

$= \boxed{(x+3)(x+2)(x-2)}$

Math 70

(11) y^3-64 difference of cubes $a^3-b^3=(a-b)(a^2+ab+b^2)$

S O AP signs

same opp always pos

$a=\sqrt[3]{y^3}=y$

$b=\sqrt[3]{64}=4$

$=\boxed{(y-4)(y^2+4y+16)}$

(12) $8x^3+27y^3$ sum of cubes $a^3+b^3=(a+b)(a^2-ab+b^2)$

S O AP signs

$a=\sqrt[3]{8x^3}=2x$

$b=\sqrt[3]{27y^3}=3y$

$=\boxed{(2x+3y)(4x^2-6xy+9y^2)}$

(13) $375q^2+3n^3q^2$

GCF $3q^2(\underbrace{125+n^3}_{\text{sum of cubes}})$ or $3q^2(n^3+125)$

$a=\sqrt[3]{125}=5$

$b=\sqrt[3]{n^3}=n$

$=\boxed{3q^2(5+n)(25+5n+n^2)}$ $\boxed{3q^2(n+5)(n^2+5n+25)}$

(14) r^6-s^6 difference of squares

$=(r^3-s^3)(r^3+s^3)$ difference of cubes & sum of cubes

$=\boxed{(r-s)(r^2+rs+s^2)(r+s)(r^2-rs+s^2)}$

OR .... harder ... diff of cubes first

$(r^2-s^2)(\underbrace{r^4+r^2s^2+s^4}_{\text{uh-oh...}})$ we know from the other

$(r-s)(r+s)(r^2+rs+s^2)(r^2-rs+s^2)$

check ...

$$\begin{array}{lllll} r^4 & -r^3s & +r^2s^2 & & \\ & +r^3s & -r^2s^2 & +rs^3 & \\ & & +r^2s^2 & -rs^3 & +s^4 \\ \hline r^4 & & +r^2s^2 & & +s^4 \end{array}$$

But we have no techniques to factor this.